Syllabus for the 2010 PCMI-UFP program

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General plan of time spent

Theory (KRV)

Main Lectures: Metrics, regularization and geometric analysis in image analysis: 15 lectures.

Extra Lectures: Geometric measure theory (GMT) with a view to data: 6 lectures. These lectures are optional for UFP participants.

Algorithms and Challenges (TJA)

Instructor: Tom Asaki will be the main lecturer for these sessions.

Lab Sessions: 15, 2.0 hour sessions: 5 projects to complete. 30 hours = 15 {0.5 hour lectures} + 15 {1.5 hour labs}

Total time

45 hours over 3 weeks. We have 3 hours a day every day. I will have another 6 hours for GMT lectures, but this will not be an official part of the UFP program, though I am hoping that many UFP participants will want to attend these lectures.

15 Lectures: Image analysis/Geometric Analysis

The Lectures

Introduction

Lecture 1 Image problems: motivation, examples.

Lecture 2 Ubiquity of metrics, Probabilistic view of ROF. L1TV, et al., brief examples, more motivation

Lecture 3 Metrics: Measuring the important and discarding the rest

Lecture 4 Regularization: close relative to metrics

Examples: a bit of a repertoire

Lecture 5 Examples 1: ROF, CE, H1, MS, and the K functional generalizations

Lecture 6 Examples 2: graph diffusion methods, non-local means, Minmal Spanning Tree-methods

Lecture 7 Examples 3: Classification Modulo Invariance (CMODI) and friends

Lecture 8 Examples 4: curve flow and level set methods.

Two Pedagogical Lectures

Lecture P9 $\int |\nabla u|^{p_1} + \lambda \int |u - f|^{p_2}$ as a playground for computation and analysis.

Lecture P10 Clustering, SVD, FLD, SVM, Concentration of measure, ...

Geometric Analysis Lectures

Lecture 11 Geometric Analysis: intro, perimeters, densities, and the coarea formula.

Lecture 12 L1TV = Multiscale Flatnorm: theory, computation and applications

Lecture 13 Covers and neighborhoods: Hausdorff measures and the Steiner-Minkowski formula.

Two more Pedagogical Lectures

Lecture P14 TV regularization and sets of finite perimeter as a path into geometric measure theory.

Lecture P15 Boundaries, level sets and distance functions: bare-handed experiments and computation.

6 Lectures: Geometric Measure Theory

The Lectures

Lecture 1 Rectifiable sets, Currents and the Flat Norm.

Lecture 2 Structure theorem for sets of finite perimeter. Federer 4.5.9 part I.

Lecture 3 Federer 4.5.9, part II.

Lecture 4 Curvature measures: sets of positive reach and beyond; $A(\epsilon)/(\epsilon)^{n-s}$ ... interesting outside of reach.

Lecture 5 Curvature measures: part II

Lecture 6 Questions and Answers: filling in gaps and some history

30 Hours: Lab projects

The Projects

We will consider challenges that cover the following five general areas of image and data analysis. Within each challenge the participant teams will work directly with data and provided software towards specific goals. These lab sessions are intended to be cooperative, interactive, stimulating and challenging.

Image Recovery from Corrupted Data. Images and data are very often "corrupted" by processes that distort or remove information. We consider corruption by noise and spatial smoothing. Our recovery tools include functional minimization denoising methods of the form "regularization + λ data fidelity" such as ROF and L^1TV and some deconvolution techniques.

Data Clustering. Certain data are meaningful when categorized into few or several collections and such clustering tools can be useful as classfiers. We will consider both classification and prediction problems. We will use clustering tools such as $k-$means, $c - means$, and EM (expectation maximization); and classification/prediction tools such as Fisher Linear Discriminant (FLD), Partial Least Squares (PLS), and Support Vector Machines (SVM).

Finding and Characterizing Boundaries. Often, the relevant information in images is the locations of object boundaries. We will explore boundary extraction and representation techniques. Our tools will include triangulation and trigonometric polynomial representations and 2d density measures.

Quantifying Shapes. Here we focus on discovering the information contained in 2d shapes. We want to consider details at all reasonable scales. Thus, our tool will be the scale-generalized Flat Norm.

Registration and Similarity Metrics. Here we consider sets of images and ask how and by how much the images differ, modulo some transformation. We consider both parametric and non-parametric transformation (registration) methods and ways to quantify image differences derived from these methods.

Additional challenges will be available for those who just can't get enough. These additional topics will include evolving curves, level set methods, segmentation and other topics.

Background Reading and Notes on References

I will not be following any particular text for the 15 lectures. Nevertheless, there are some references that go well with the course. The background necessary for fully appreciating the lectures is a graduate course in analysis, some exposure to PDE and variational analysis and a sense for "how things go" geometrically. If you are concerned about background, you might:

1. Review your graduate analysis class,
2. read the review paper by Chan, Shen and Vese [8] and peruse Tony Chan and Jackie Shen's book [7]. Then find papers on the UCLA CAM website [26] on the pieces that peak your interest (or you could simply go to the papers listed below after the heading **Specific Papers**), and then
3. lightly peruse Jack Lee's *Smooth Manifolds* [18]. It is very well written, though some find it too wordy. But it is a fast enough read that it doesn't matter and anyway, the illustrations and detail are very helpful for a first exposure. Note: this book contains far more than is necessary for full comprehension of the summer school lectures. Thus the "lightly peruse" above.

Background for the Lab Sessions: the same as for the lectures, except that you should know how to use matlab. **Ideally:** you will bring your own laptop to the summer school with your own copy of Matlab and the (1) Image Processing, (2) Statistics, (3) Optimization, (4) Global Optimization, (5) Signal Processing toolboxes.

For the more (overly) ambitious student, here is a more extensive listing of references. Note that much of this goes way beyond what we will cover – it is simply further along the same direction. The last two entries –**Specific Papers** and **Other** – are probably the first place to start if you intend to study something from the list below. If you want to attend my GMT lectures, looking at the first 5 chapters of Frank Morgan's book ahead of time will be helpful.

- **Image Analysis:** Tony Chan and Jackie Shen's book [7] is best for it's choice of topics. The papers at the UCLA CAM preprint server [26] are a great resource. There are of course many other books on image processing and analysis, some quite good. One such reference is Sapiro's book [29] which I have used, at various times, with benefit.
- **Analysis Background:** Any graduate course in analysis is more than enough. Folland's book [15] is my own standard for a first graduate course in analysis. Evans and Gariepy's [13] wonderful monograph is something I would encourage everyone to study through. I will not assume the level of either for the first 15 lectures. The GMT

lectures will be exploring advanced concepts, similar to Evans and Gariepy in level, but with a distinct emphasis on intuition, geometry.

- **PDE:** At this level, I like Evans' PDE book [12]. Even the appendix is a superb reference for the most used parts of several subjects.
- **Variational Analysis:** Ekeland and Temam [11] is a reference I have used fairly often.
- **Level set methods:** Osher and Fedkiw [25] is a good reference, as are the many papers you can find on the CAM preprint server [26] at UCLA.
- **Geometric Measure Theory:** Frank Morgan's book [21] is a standard first, and fairly quick, introduction to geometric measure theory. For more details, Leon Simon's text [30] or Krantz and Parks [17] are both good. Evans and Gariepy [13] is great for what it covers. Mattila's book [20] is also very valuable for the parts of GMT not involving currents. The ultimate reference for a fair bit of material is Federer's famous tome [14], but it is slow going and not recommended as the first exposure. (In fact, Morgan wrote his book as an interface to Federer.) There is another book by Lin and Yang [19] that has the right topics (and a fair number of typos that can make it difficult reading for non-experts).
- **Specific Papers:** Rudin-Osher-Fatemi (ROF) TV regularization paper [28], Mumford-Shah inspired segmentation papers [32, 31], Level sets [24, 23] – read the paper from 2000 first, the L1TV functional [6], L1TV computes the flatnorm [22, 34], metrics [33], Clasification Mod invariance with face recognition application [16], graph based image denoising [1], diffusion geometry [9, 10, 27], non-local means denoising [4], image inpainting [3, 2], Chambolle's ROF algorithm [5], a review article on variational PDE methods for images [8].
- **Other:** 2005 IPAM summer school lectures: http://www.ipam.ucla.edu/programs/gss2005/.

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