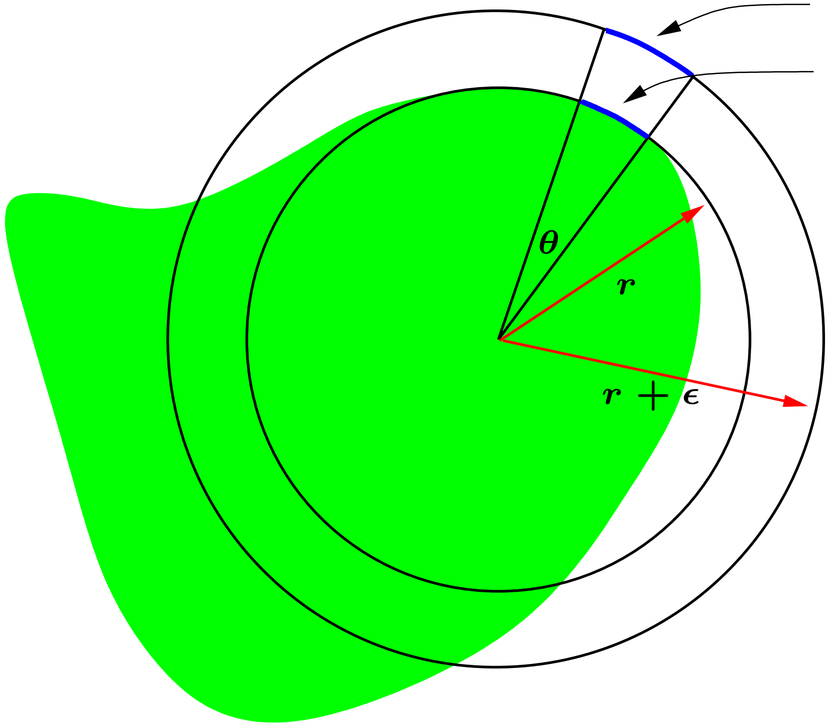
Tubular Formula: A quick proof for nice cases

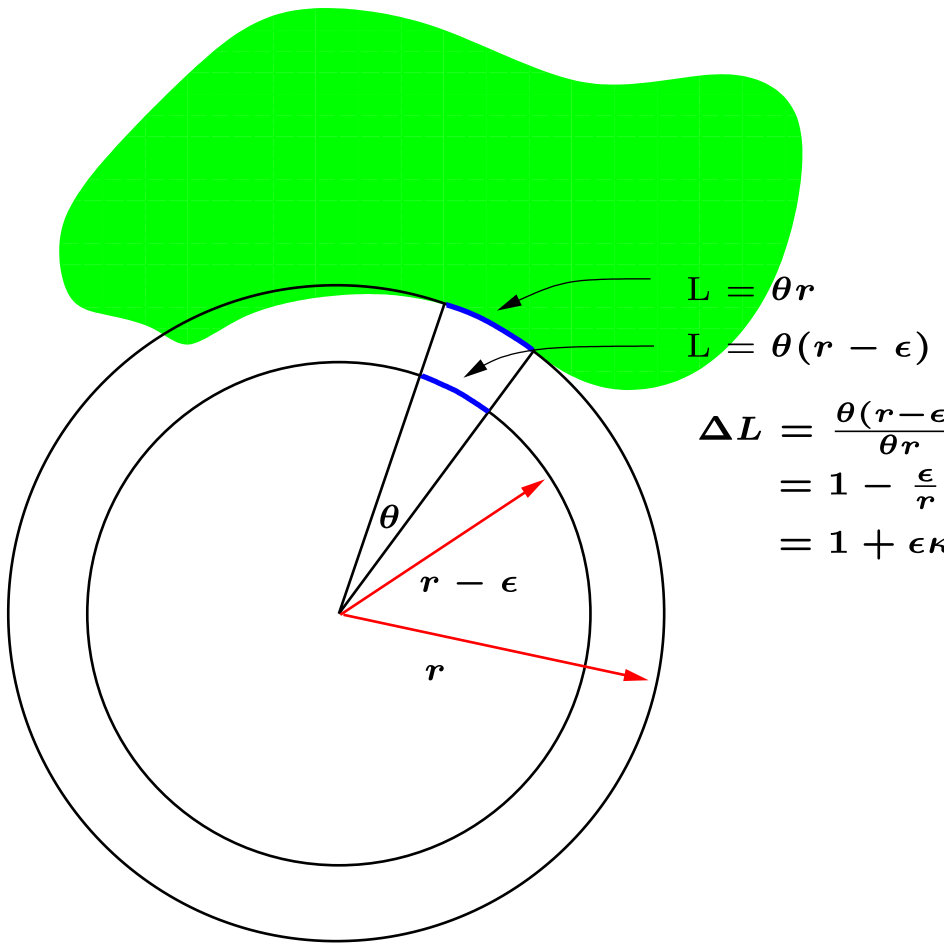


$\mathrm{L} = \theta(r + \epsilon)$

$\mathrm{L} = \theta r$

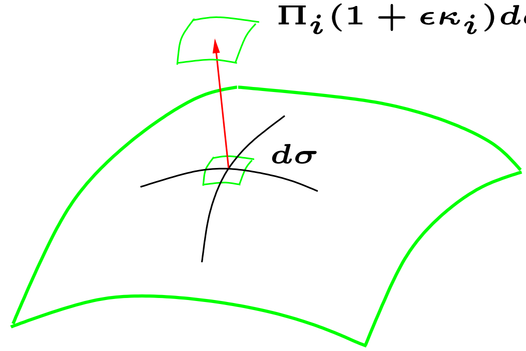
$$\Delta L = \frac{\theta(r+\epsilon)}{\theta r} = 1 + \frac{\epsilon}{r} = 1 + \epsilon\kappa$$

Tubular Formula: A quick proof continued



$$\Delta L = \frac{\theta(r-\epsilon)}{\theta r}$$
$$= 1 - \frac{\epsilon}{r}$$
$$= 1 + \epsilon\kappa$$

Tubular Formula: A quick proof concluded



$$\begin{aligned}
\mathcal{H}^{n-1}(\partial(W+B_\epsilon)) &= \int_{\partial W} \Pi_{i=1}^{n-1}(1+\epsilon\kappa_i)d\mathcal{H}^{n-1} \\
&= \int_{\partial W} 1 d\mathcal{H}^{n-1} + \epsilon \int_{\partial W} \sum_i \kappa_i d\mathcal{H}^{n-1} \\
&+ \ldots + \epsilon^k \int_{\partial W} \sum_{s\in S(k)} \Pi_{i\in s}\kappa_i d\mathcal{H}^{n-1} \\
&+ \ldots + \epsilon^{n-1} \int_{\partial W} \Pi_{i=1}^{n-1}\kappa_i d\mathcal{H}^{n-1}
\end{aligned}$$

Tubular Formula: A quick proof concluded – really

$$
\begin{aligned}
\mathcal{H}^n(W + B_r) &= \mathcal{H}^n(W) + \int_{\epsilon=0}^{\epsilon=r} \mathcal{H}^{n-1}(\partial(W + B_\epsilon))d\epsilon \\
&= \mathcal{H}^n(W) + r\int_{\partial W} 1 d\mathcal{H}^{n-1} + \frac{r^2}{2}\int_{\partial W} \sum_i \kappa_i d\mathcal{H}^{n-1} \\
&+ \ldots + \frac{r^{k+1}}{k+1}\int_{\partial W} \sum_{s \in S(k)} \Pi_{i \in s} \kappa_i d\mathcal{H}^{n-1} \\
&+ \ldots + \frac{r^n}{n}\int_{\partial W} \Pi_{i=1}^{n-1} \kappa_i d\mathcal{H}^{n-1} \\
&= V^n(W) + \sum_{i=1}^{n} \frac{\epsilon^i}{i} V_{n-i}(W)
\end{aligned}
$$