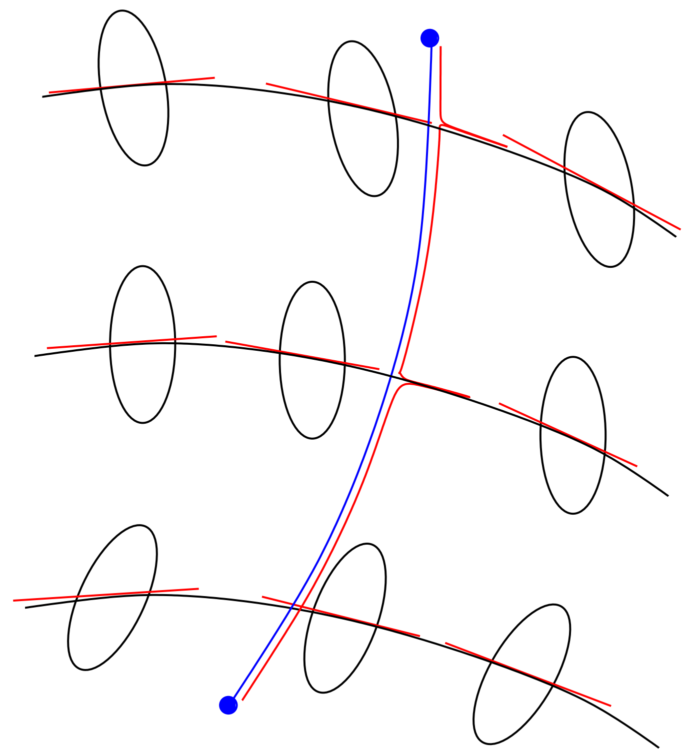
Ignoring the Unimportant: Null Riemannian Metrics

Next step: singular metric fields.



The idea: Replace $M \rightarrow PP^T M PP^T$ where PP^T is a projection annihilating ignored directions.

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Outline of where we are with *null Riemannian Geometry*:

- Regularize:

$$M_\epsilon = M - (1 - \epsilon) P_N P_N^T M P_N P_N^T \tag{1}$$

  or

$$M_\epsilon = P_N P_N^T M P_N P_N^T + \epsilon I \tag{2}$$

- It is straightforward to prove that on compact manifolds, the resulting distances (the lengths of geodesics) converge under fairly mild conditions. I.e.

$$\rho_{M_\epsilon} \to_{\epsilon \to 0} \rho_M. \tag{3}$$

  The converse is of course false: there are distance functions which converge but whose underlying metric field does not (oscillations are enough). Geodesics are wildly non-unique of course.

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- The relation to singular Riemannian geometry: some details here.
  - Singular Riemannian Geometry: introduced into control theory by R. Brockett, asks for shortest paths between two points in which the geodesic tangents are restricted to a k-distribution H. To be useful, iterated Lie Brackets must fill the tangent space. (H satisfies the Hormander condition.)
  - Example: Navigating around a plane on a tricyle. The state space is 3 dimensional (counting 2-d position and orientation of tricyle) but the controls access two dimensional subapces of each tangent space.
  - Relation of singular Riemannian geometry to null Riemannian geometry?

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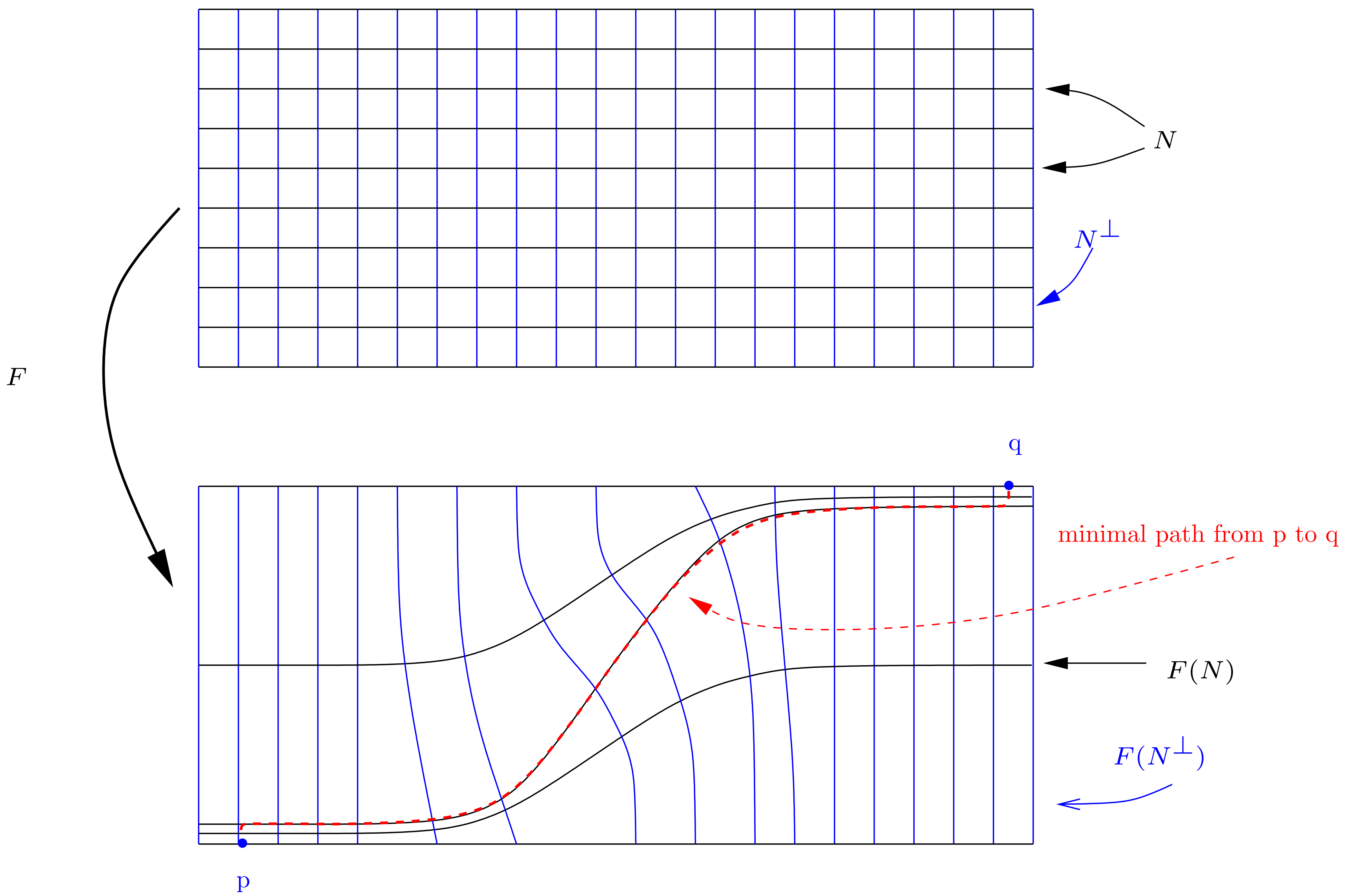

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  - Rough Conjecture: minimal length paths exist which are everywhere in N or $N^{\perp}$, where N is the null distribution.

- Algorithms: under construction.

- What problems are we developing the algorithms on?

  - Problem: distances on graphs $\rightarrow$ variation on graphs.

$$M(x) = \nabla f \nabla f^T + I \rightarrow M_\epsilon(x) = \nabla f \nabla f^T + \epsilon I \tag{4}$$

  - Problem: distances in (smoothed) image spaces modulo translation
  - Problem: distances in state spaces modulo time shifts: The idea is that the metric is obtained with pullback from the initial time $t = 0$ to the present time $t = T$ and then the vector field directions define the null directions.