184 + missing) now fixed in notes sent out!

$$|h| \equiv \int |h| + \left(\int |\nabla h|^2\right)^{1/2} + \int |\Delta h|$$

$$\left(\int |\nabla h|\right)^2 \leq |h|^2$$

$$r(h) = \int |\nabla h|^2$$

$$\int |\nabla h| \qquad \sqrt{\int |\nabla h|^2}$$

Fix 8.2.1

$$|h| \equiv \int_\Omega |h(x)|\,dx + \left(\int_\Omega |\nabla h(x)|^2 dx\right)^{1/2} + \left(\int_\Omega |\Delta h(x)|^2 dx\right)^{1/2}$$

$$f \ni \left(\int f^2\right)^{1/2} < \infty$$

$$\left(\int f \cdot f\right)^{1/2} < \infty$$

$$|\alpha f| = |\alpha| \, |f|$$

$f, g \in L^2(\Omega; \mathbb{R})$

$$\int_\Omega f g \, dx$$

$\forall \xi : \Omega \to \mathbb{R}^n$

$$\left(\int_\Omega |f|^2 \, dx\right)^{1/2}$$

$f, g \in L^2(\Omega; \underset{\in \mathbb{R}^n}{\mathbb{R}^n})$

$$\left(\int f^2\right)^{1/2}$$

$$\left(\int f \cdot f\right)^{1/2}$$

$$\underbrace{F(u) = \int_\Omega \nabla u \cdot \nabla u \, dx}_{\text{linear approx}} \qquad F : L^2(\Omega, \mathbb{R}) \to \mathbb{R}$$

h

$$\nabla F(u) = \Delta u$$

$$\int_\Omega \Delta u \; h \, dx$$

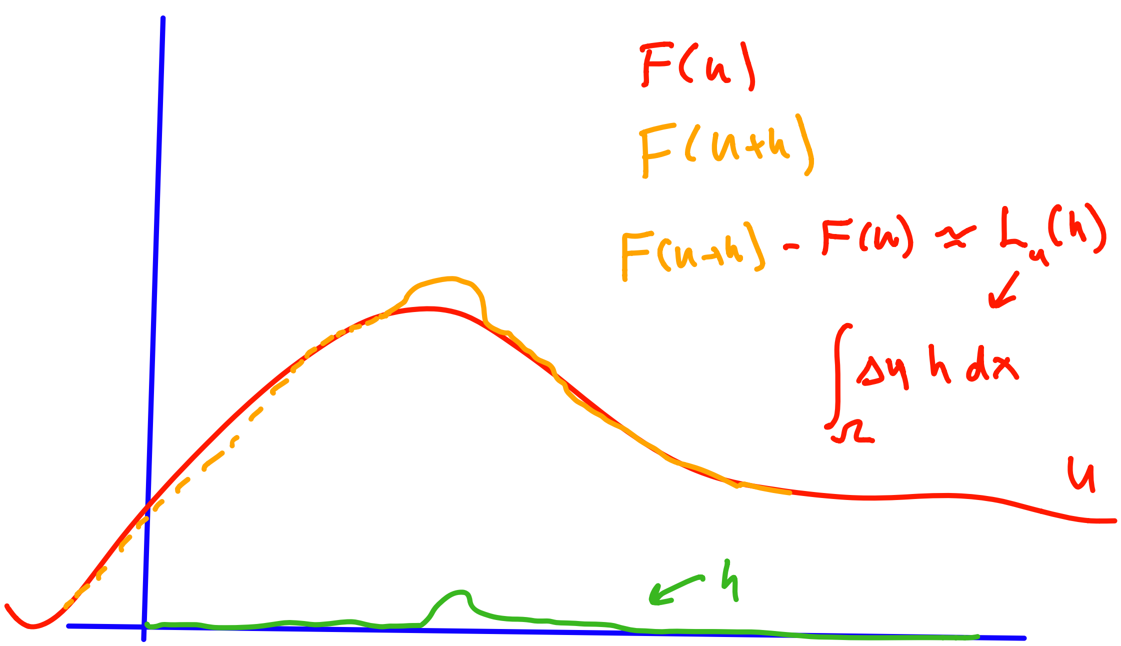
$\Omega \subset \mathbb{R}^n$

$F(u)$

$F(u+h)$

$F(u+h) - F(u) \approx L_u(h)$

$$\int_\Omega \Delta u \, h \, dx$$



$$F(u+h) - F(u) = L_u(h) + \underbrace{g(h)}_{o(h)}$$

linear operator

"little o" of h

$$\frac{|g(h)|}{|h|} \to 0 \qquad |h| \to 0$$

Euler - Lagrange

Banach fixed point theorem question:

$$X \xrightarrow{F} X$$

complete normed vector space

$$|\ |_X \qquad |\ |_X$$

The norm, with subscript (usually supressed) showing which space the norm is from.

$$F(x) = x$$

looking for x that makes this equation true.

If

assume

$$|F(x) - F(y)| \le K|x-y|$$

$$0 \le K < 1$$

* contraction condition *

$$\exists! \; x^* \ni F(x^*) = x^*$$

then: this is true

$$x_0$$

$$x_1 \equiv F(x_0)$$

$$x_2 \equiv F(x_1)$$

$$\vdots$$

$$x_{n+1} \equiv F(x_n)$$

converges to x^*

$$|x_{n+1} - x_n| = |F(x_n) - F(x_{n-1})|$$

$$\le K|\underline{x_n - x_{n-1}}|$$

$$|x_{n+1} - x_n| \le K^n |x_1 - x_0|$$

$$|x_{m+1} - x_\ell| \le K^n |x_1 - x_0| + K^{n-1} |x_1 - x_0| + \cdots K^\ell |x_1 - x_0|$$

$m+1 > \ell$

$$= K^\ell \left(\underbrace{K^{m-\ell} + \cdots + 1} \right) |x_1 - x_0|$$

$$\le K^\ell \cdot \frac{1}{1-K} |x_1 - x_0|$$

Focus again on 8.2, 8.3 and exercise 8.2.1 (I will correct the error in students noted in class).