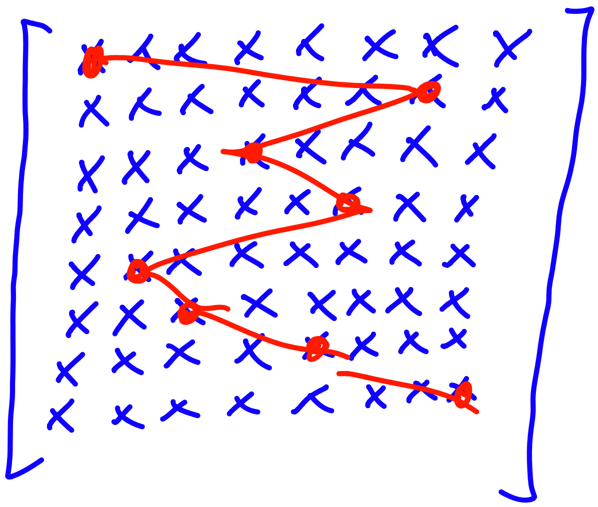
Question on 6.1.12.1



$$\underbrace{1\ 7\ 4\ 6\ 2\ 3\ 5\ 8} \leftarrow \underline{(-1)^m}$$

$(12)(17)($

8!

$\text{sign}(\sigma)\ a_{11} \cdot a_{27} \cdot a_{34} \cdot a_{46} \cdot a_{52} \cdot a_{63} \cdot a_{75} \cdot a_{88}$

$L : \mathbb{R}^n \to \mathbb{R}^n$

$\det(L)$ = volume of the image of the unit cube under L.

$\mathrm{vol}^n(L(\text{C}))$

Derivative

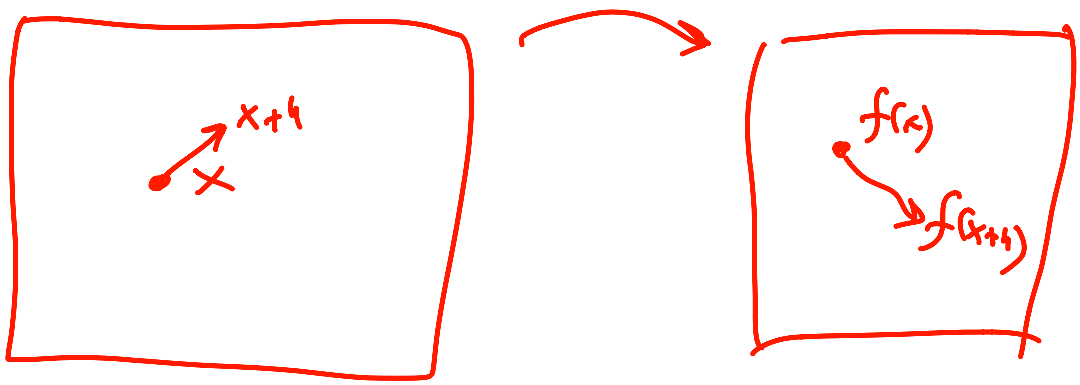
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

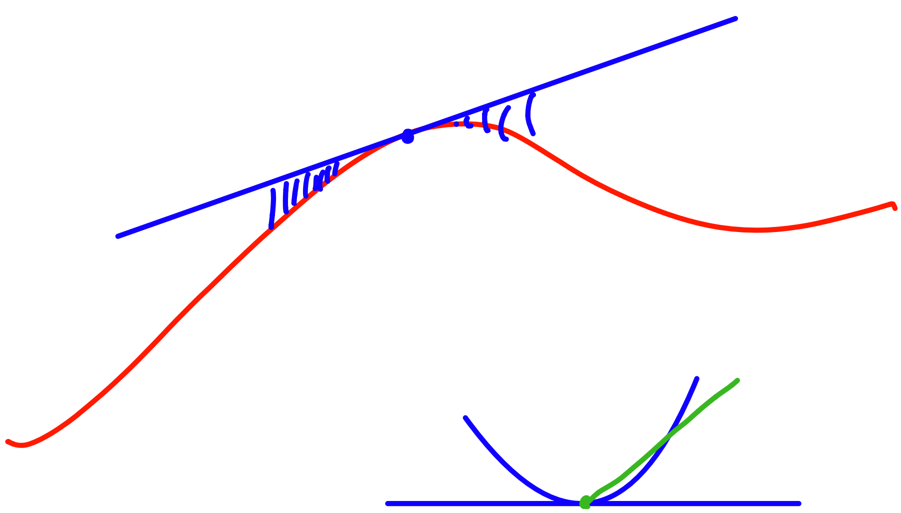
x fixed

$$f(x+h) - f(x) = A(h) + \underbrace{g(h)}_{o(h)}$$

$$\boxed{\Delta_x f(h) - A(h) \sim o(h)}$$

$$\frac{|g(h)|}{|h|} \to 0 \quad |h| \to 0$$





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$$F(u) \equiv \int_\Omega \nabla u \cdot \nabla u \, dx$$

$$\underline{F(u+h) - F(u)}$$

$u : \Omega \subset \mathbb{R}^n \to \mathbb{R}$

$h : \Omega \subset \mathbb{R}^n \to \mathbb{R}$

$h = 0$ on $\underbrace{\partial\Omega}_{\text{boundary of } \Omega}$

$$\int_\Omega \nabla(u+h) \cdot \nabla(u+h) - \int \nabla u \cdot \nabla u$$

$$\int_\Omega \nabla u \cdot \nabla u + 2 \nabla u \cdot \nabla h + \nabla h \cdot \nabla h$$

$$h = \underline{\alpha_n g}$$

$$\int_\Omega \cancel{\nabla u \cdot \nabla u} + 2\alpha_n \nabla u \cdot \nabla g + \alpha_n^2 \nabla g \cdot \nabla g$$

$$- \int_\Omega \cancel{\nabla u \cdot \nabla u}$$

$$\int_\Omega 2\alpha_n \nabla u \cdot \nabla g + \alpha_n^2 \nabla g \cdot \nabla g \, dx$$

If we pick $|g| = 1$ $\qquad |h|^2 C$

$$|g| = 1$$
$$\Downarrow$$
$$|\alpha_h g| = |\alpha_h|\,|g| = |\alpha_h|$$

$$\underbrace{2\int_\Omega \alpha_h \nabla u \cdot \nabla g \, dx}_{\text{Linear operator on } h} + o(h)$$

$$(uv)' = u'v + uv'$$

$$\int_a^b (uv)' = \int_a^b u'v + \int_a^b uv'$$

$$uv\Big|_a^b = \underbrace{\qquad}_{\text{" "}}$$

$$g : \mathbb{R}^n \to \mathbb{R}$$

$$v : \mathbb{R}^n \to \mathbb{R}^n$$

$$\boxed{\nabla \cdot (g v)}$$

$$\nabla \cdot w$$

$$\frac{\partial w_1}{\partial x_1} + \frac{\partial w_2}{\partial x_2} + \cdots$$

$$\left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}, \cdots, \frac{\partial}{\partial x_n} \right)$$

$$(w_1, w_2, w_3, \ldots, w_n]$$

$$\frac{\partial}{\partial x_1}(g v_1) + \frac{\partial}{\partial x_2}(g v_2) + \cdots +$$

$$\frac{\partial}{\partial x_n}(g v_n)$$

$$g_1 \equiv \frac{\partial}{\partial x_1} g$$

$$g_1 v_1 + g_2 v_2 + \cdots$$

$$+ g \nabla \cdot v$$

$$\underbrace{\int_\Omega \nabla \cdot gV}_{\shortparallel} = \int_\Omega \nabla g \cdot V + \int_\Omega g\, \nabla \cdot V$$

$$\int_{\partial\Omega} gV \cdot \vec{n}\, d\sigma = \int_\Omega \nabla g \cdot V + \int_\Omega g\, \nabla \cdot V$$

$$\underbrace{2 \int_\Omega \alpha_h \nabla u \cdot \nabla g \; dx}_{} \qquad V \equiv \nabla u$$

$$= 2\alpha_h \left[\int_{\partial\Omega} g \nabla u \cdot \vec{n}\, d\sigma - \int_\Omega g \Delta u \right]$$

$$= \boxed{-2 \int_\Omega \Delta u \cdot h \; dx}$$

linear operator on h

- Exercise 8.2.1
- Read/study 8.3