Regular Level sets for $f: E \subset \mathbb{R}^n \to \mathbb{R}^m$

$$k \equiv \max(n-m, 0)$$

$$X_c \equiv \{x \mid f(x) = c\} \qquad c \in \mathbb{R}^m$$

X_c is <u>regular</u> if $\forall\ y \in X_c$

and some small enough ε

$$B(y,\varepsilon) \cap X_c \cap E$$

$$\simeq B(y,\varepsilon) \cap \{y + V_y\} \cap E$$

<u>Examples</u>

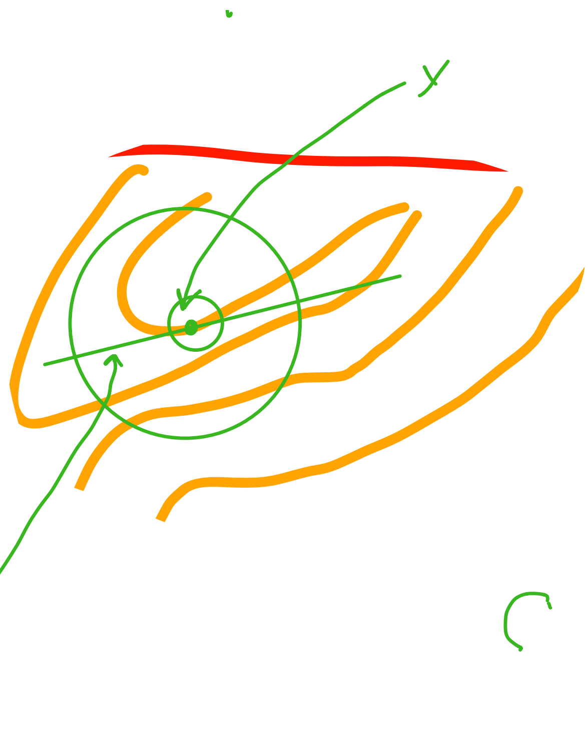
$m = 1$

$n = 2$

$E = \mathbb{R}^2$

$k = \max(n-m, 0)$

$= 1$

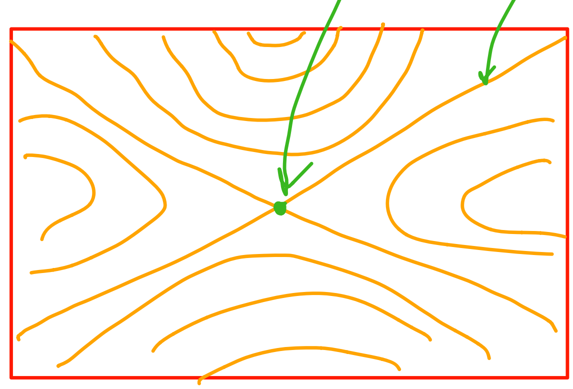


$V_y + x$

Example of Level set that is <u>not</u> regular

this $y \in X_c$ has no approx ~

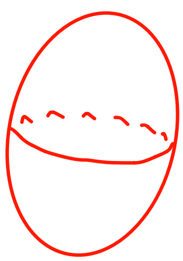
not regular X_c

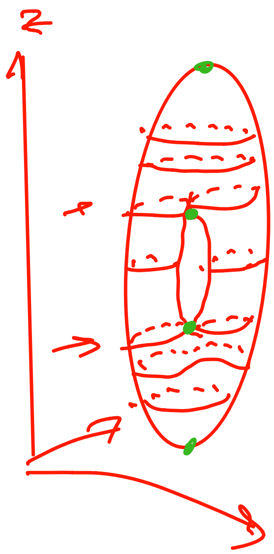


$$\boxed{F(x+h)-F(x) = Ah + \underbrace{g(h)}_{o(h)}}$$

*

John Milnor : Morse Theory





$f : T \to z$

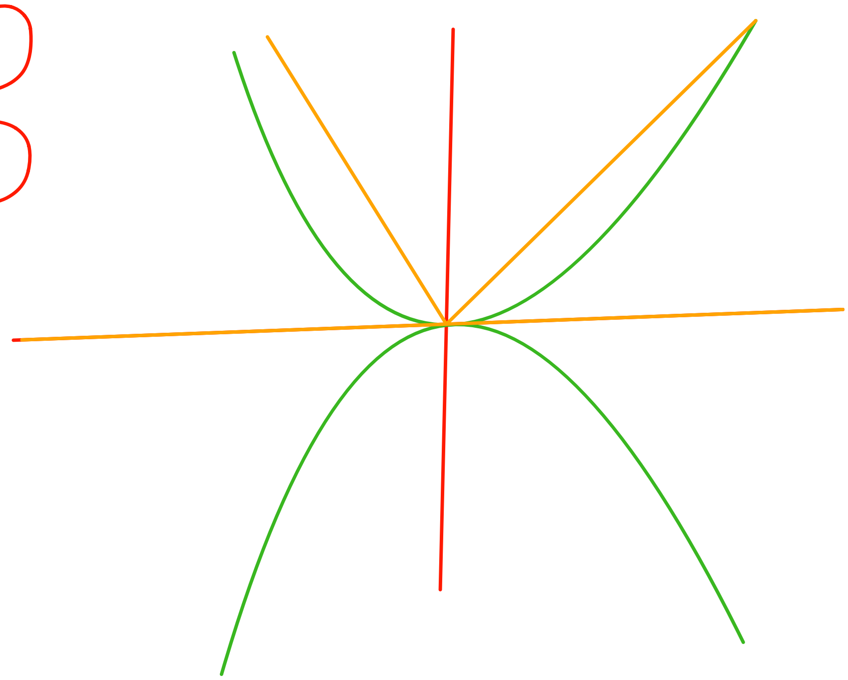
Osher + Sethian   Level set method

cited > 12000 times

Ex 8.3.2

1

2



$f(x) = x^2$ x rational

$f(x) = -x^2$ x irrational

not diff anywhere $\left\{ \begin{array}{ll} \hat{f}(x) = |x| & x \text{ rational} \\ \hat{f}(x) = 0 & x \text{ irrational} \end{array} \right.$

8.3.3

$$f(x) \equiv \sum_{i \ni q_i < x} \frac{1}{2^i}$$

this works!!

y is irrational

$$|y - q_i| > 0 \quad \forall i$$

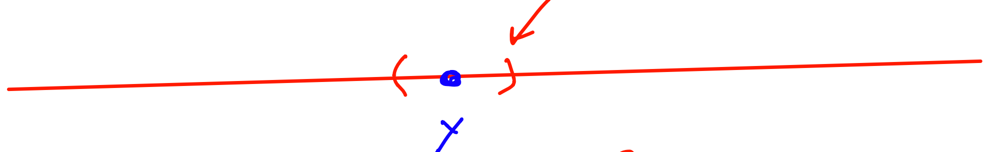
$$\underset{\text{finite}}{E} \subset \{q_i\}_{i=1}^{\infty}$$

$$0 < \sup_{q_i \in E} \left|\frac{1}{2^i}\right| = \frac{1}{2^K} \qquad K = \max i \ni q_i \in E$$

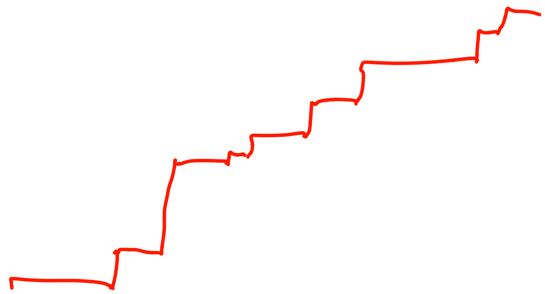
$$E_K = \{q_i\}_{i=1}^{K}$$

$$f \text{ jumps} < \frac{1}{2^K} \quad \forall \; E^c$$

$$\delta \equiv \min_{i=1\ldots K} |y - q_i| \qquad |x-y| < \delta$$

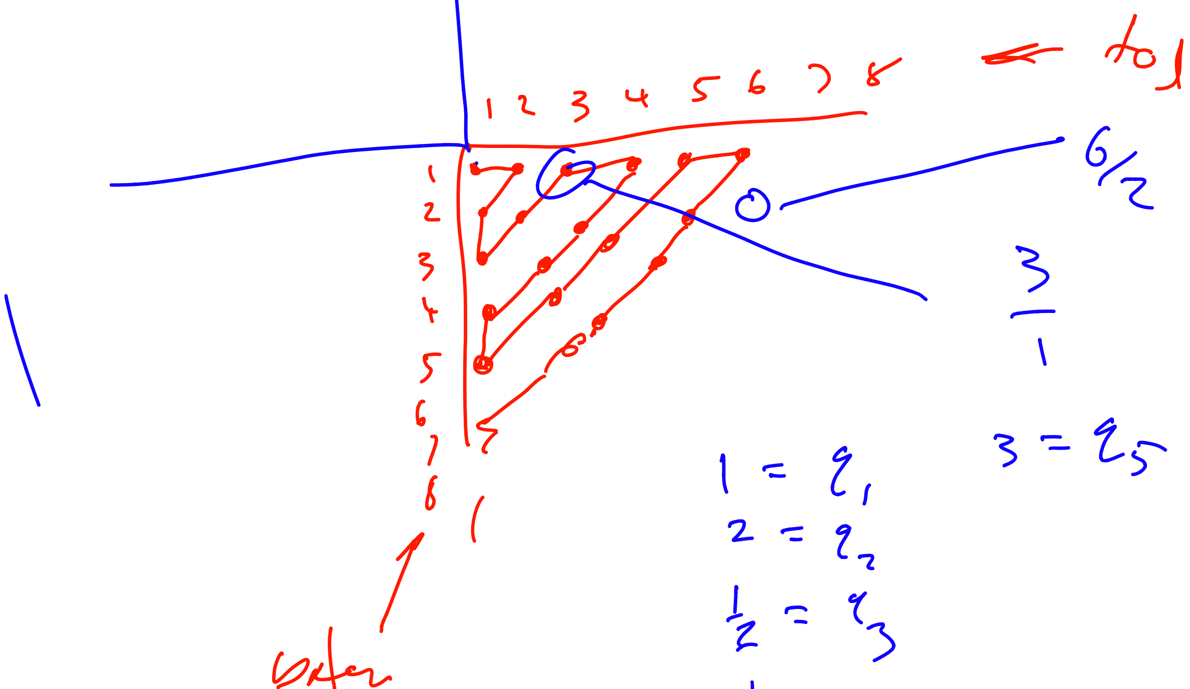


$$\sum_{i=K+1}^{\infty} \frac{1}{2^i} = \frac{1}{2^K}$$



$\frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{8} \cdot \frac{1}{16}$

Q countable $\quad Q = \{q_i\}_{i=1}^{\infty}$



$\frac{3}{1}$

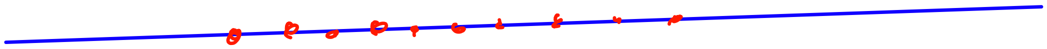
$3 = q_5$

$1 = q_1$

$2 = q_2$

$\frac{1}{2} = q_3$

$\frac{1}{3} = q_4$



$$\mathbb{Q} \subset \bigcup B\left(q_i, \frac{\varepsilon}{2^i}\right)$$

$$\Rightarrow \mu(\mathbb{Q}) < \sum \frac{\varepsilon}{2^i} = \varepsilon$$

$$\mu(\mathbb{Q}) < \varepsilon \quad \forall \varepsilon \Rightarrow$$

$$\Rightarrow \mu(\mathbb{Q}) = 0$$