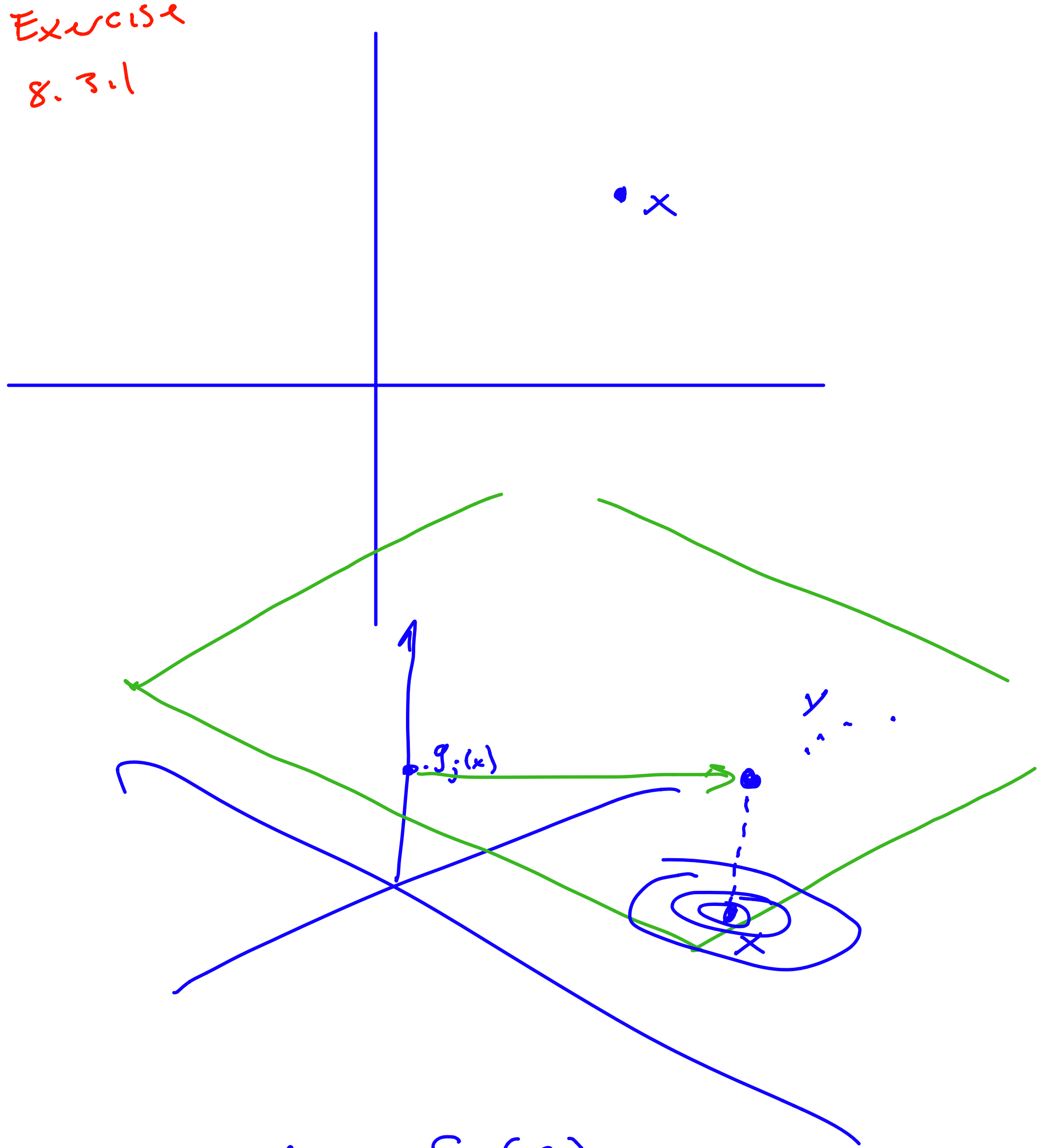
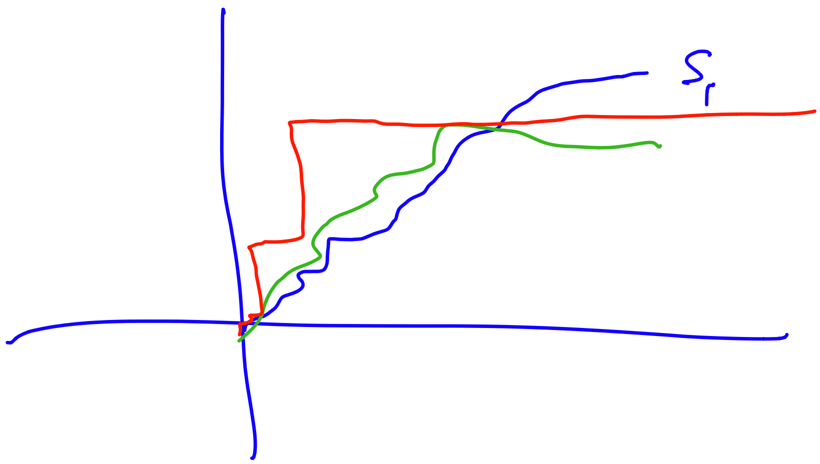
on Exercise 8.3.1



$$|g_j(y) - g_j(x)| < \delta_j(\varepsilon) \equiv \sup_{y \in B(x,\varepsilon)} |g_j(y) - g_j(x)|$$

$$\delta_j \to 0$$
$$\varepsilon \to 0$$



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1. $$F(x+h) - F(x) = \underbrace{A\,h}_{D_xF\,h} + \underbrace{g(h)}_{o(h)}$$

$D_xF(h)$

$$\frac{|g(h)|}{|h|} \to 0 \qquad |h| \to 0$$

2. what this looks like when

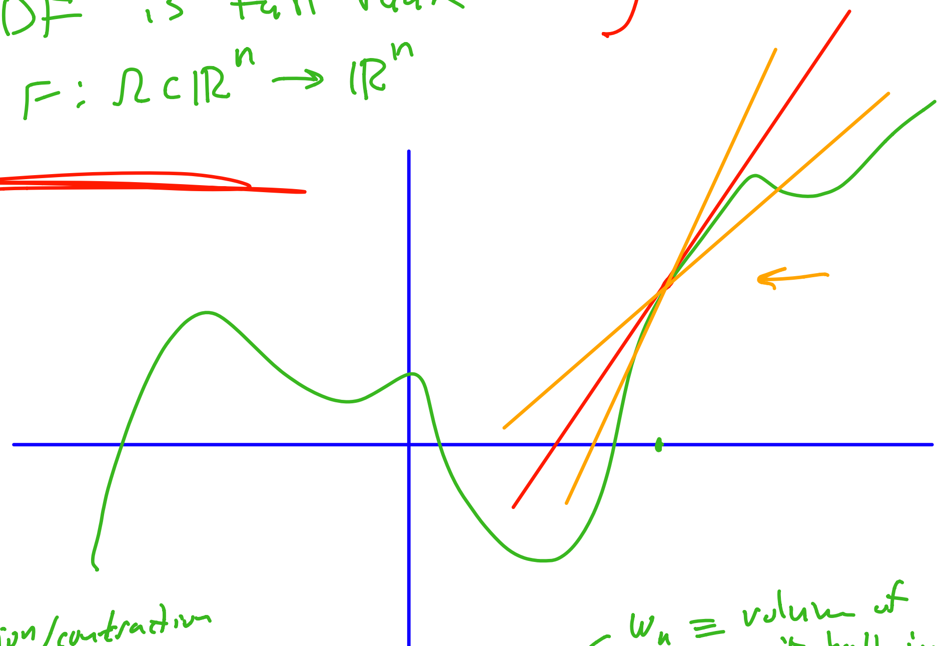
   (a) $F : (\text{function space}) \to \mathbb{R}$

   (b) $F : \mathbb{R}^n \to \mathbb{R}$ ← ∇F row vector $1 \times n$ matrix

   (c) $F : \mathbb{R}^n \to \mathbb{R}^m$

(3) F approx by $DF \equiv A$

- $F \in C^1(\Omega)$
- DF is full rank
- $F : \Omega \subset \mathbb{R}^n \to \mathbb{R}^n$



Volume expansion/contraction

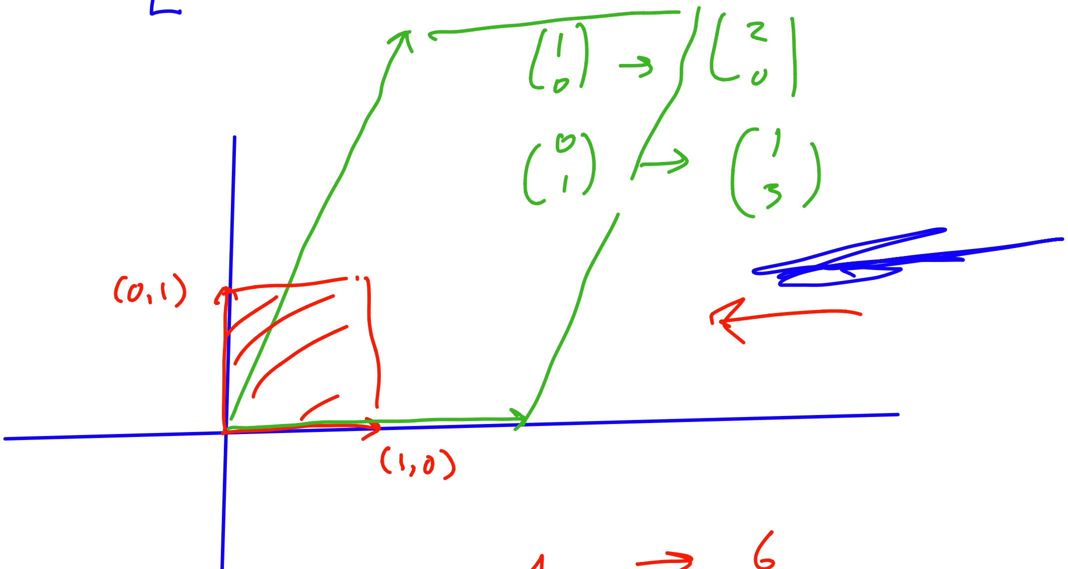
$\omega_n \equiv$ volume of unit ball in $\mathbb{R}^n$

$$\mathrm{Vol}^n(F(B_x(\varepsilon))) \approx \underbrace{|\det(D_x F)|}_{\text{Volume dilation factor}} \underbrace{\omega_n \varepsilon^n}_{\substack{\text{volume of } B_x(\varepsilon) \\ \shortparallel \\ B(x, \varepsilon)}}$$

$(F : \mathbb{R}^n \to \mathbb{R}^n)$

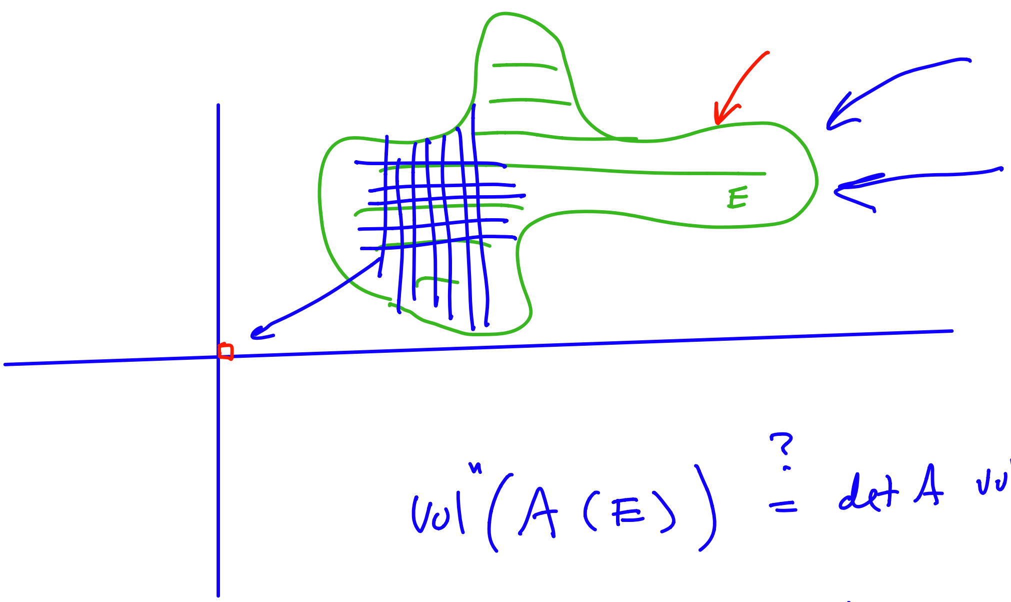
$$\frac{\mathrm{Vol}^n(F(B_x(\varepsilon)))}{\omega_n \varepsilon^n} \xrightarrow[\varepsilon \to 0]{} |\det(D_x F)|$$

$$\begin{bmatrix}2 & 1\\ 0 & 3\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} \to \begin{bmatrix}w\\ z\end{bmatrix}$$



$$1 \to 6$$

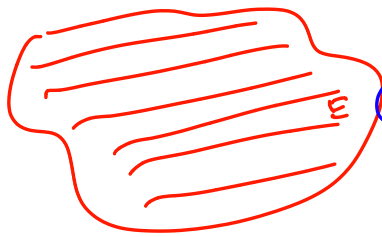
$$\det\left(\begin{bmatrix}2 & 1\\ 0 & 3\end{bmatrix}\right) = 6 \ !$$



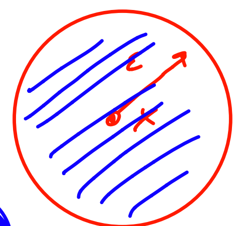
$$\mathrm{vol}^n(A(E)) \overset{?}{=} \det A \ \mathrm{vol}^n(E)$$

$$\mathrm{vol}^n(A(c_i + y_i)) = V^n(A(c_i))$$

$\mathcal{L}^2(E) > 0$



for almost every $x \in E$



χ_E

$$\chi_E(x) = \lim_{\varepsilon \to 0} \frac{\int_{B_\varepsilon} \chi_E}{\omega_n \varepsilon^n}$$

$$\frac{1}{1-a} = 1 + a + a^2 + a^3 + a^4 + \cdots$$

$$-1 < a < 1$$

$$\boxed{(I - A)^{-1} = I + A + A^2 + A^3 + \cdots}$$

$$|A| < 1$$

$$\dot{x} = Ax \rightarrow e^{At} x_0$$

$$\dot{x} = ax \rightarrow x_0 e^{at}$$

$$1$$

$$I + At + \frac{1}{2} A^2 t^2 + \frac{1}{3!} A^3 t^3 + \cdots$$

$$f : \mathbb{R}^n \rightarrow \mathbb{R}^n$$

x $\qquad \overbrace{D_x f \quad \text{nonsingular}}$

$f \in C^1$ $\leftarrow$ $\qquad e^{-\frac{x^2}{\sigma^2}}$

f is invertible at x

Read 8.4

Do 8.3.2 – 8.3.4