From the Book

$$f(x+h) - \sum_{k=0}^{n} f^k(x)\frac{h^k}{k!} = f^{n+1}(c(h))\,\frac{h^{n+1}}{(n+1)!}$$

for some $c(h)$ between x and $x+h$.

Starting with Questions

should be min(n,m)

fix this

Area/Coarea Formulas: $f : \mathbb{R}^n \to \mathbb{R}^m$

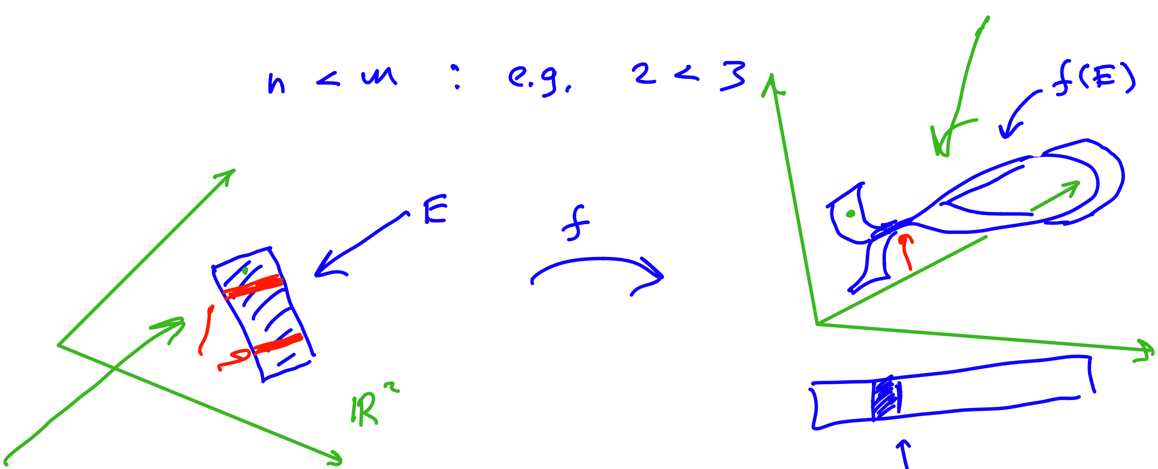
$$\int_\Omega g(x) J^* f\,dx = \int_{f(\Omega)} \left(\int_{f^{-1}(w)} g(x)\, d\mathcal{H}^{\max(n-m,0)}(x) \right) d\mathcal{H}^m(w)$$

→ a very powerful general tool for tracking and computing mapped volumes

→ We encounter Outer Measures and Hausdorff Measures in earnest here!

Explain this

$n < m$: e.g. $2 < 3$



from above f(E) looks like this, with the solid blue bar the part that coincides with another piece

in case $g(x) = 1$

$f : \mathbb{R}^2 \to \mathbb{R}^3$ as

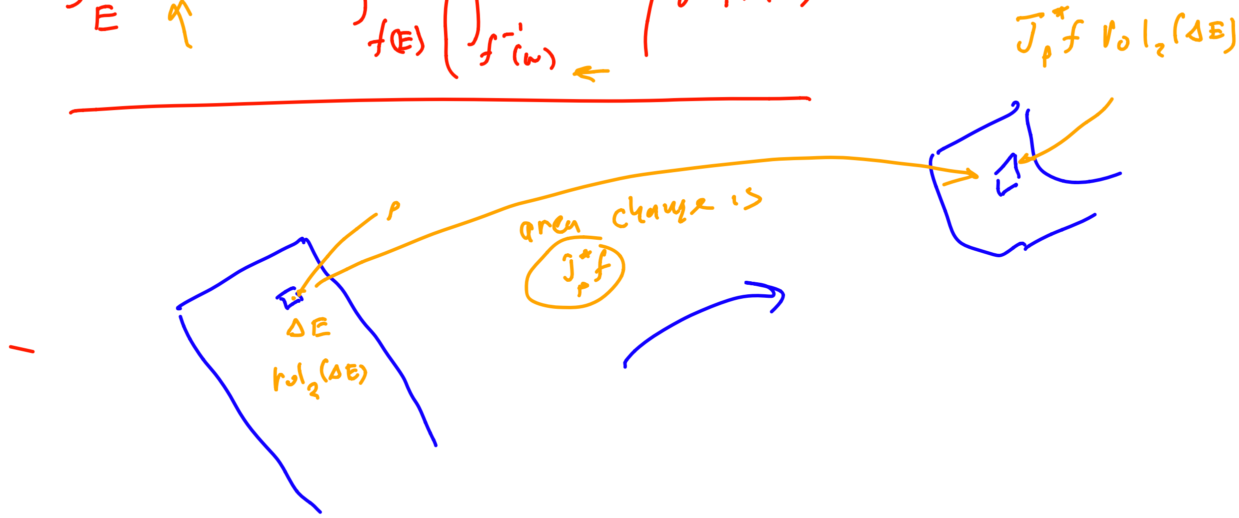
does what is shown

then

$$\int_E J_x^* f \, dx = \int_{f(E)} \left(\int_{f^{-1}(w)} d\mathcal{H}^0(x) \right) d\mathcal{H}^2(w)$$

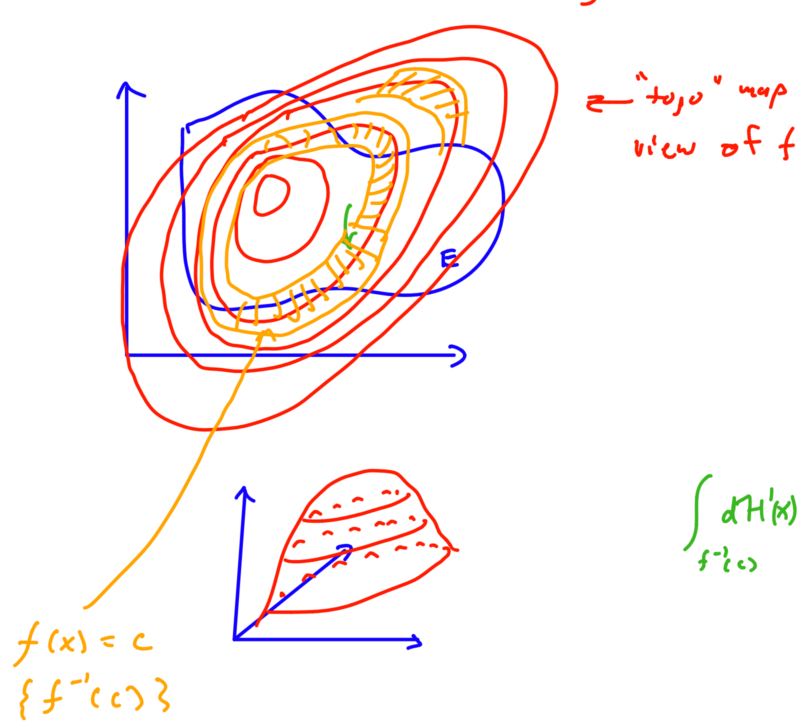
this gives me an integer at every point in $f(E)$

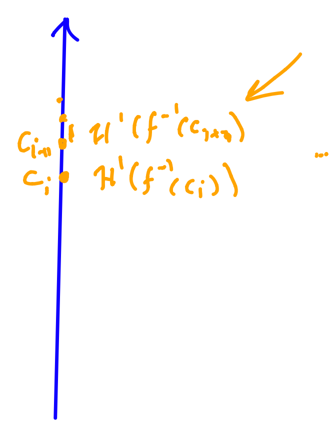
of $f(E)$ ... i.e. the two red pieces in E



$n > m$ $\qquad$ $n = 2$, $m = 1$

$f : \mathbb{R}^2 \to \mathbb{R}$





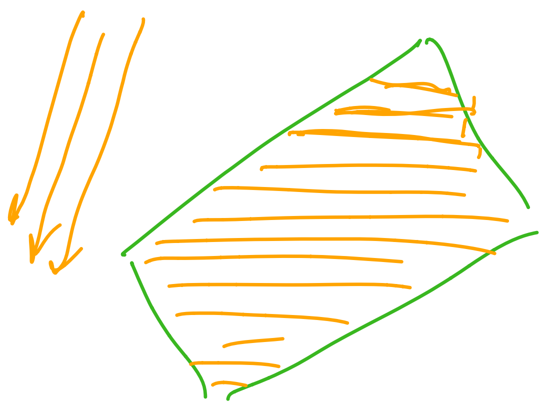
$$\int_{f^{-1}(c)} d\mathcal{H}^1(x) = \text{length of } f^{-1}(c)$$

$$J^m f = \sqrt{\nabla f^T \cdot \nabla f} = |\nabla f|$$

$n > m$

$$\sqrt{\det(Df \cdot Df^T)}$$

$$\int_E |\nabla f| \, dx = \int_{f(E)} \underbrace{\left(\int_{f^{-1}(y)} d\mathcal{H}^1_x \right)}_{\text{length of } f^{-1}(y)} d\mathcal{H}^1_y$$



$$f : \mathbb{R}^2 \to \mathbb{R}$$

$$z = f(x, y) = x^2 + y^2$$