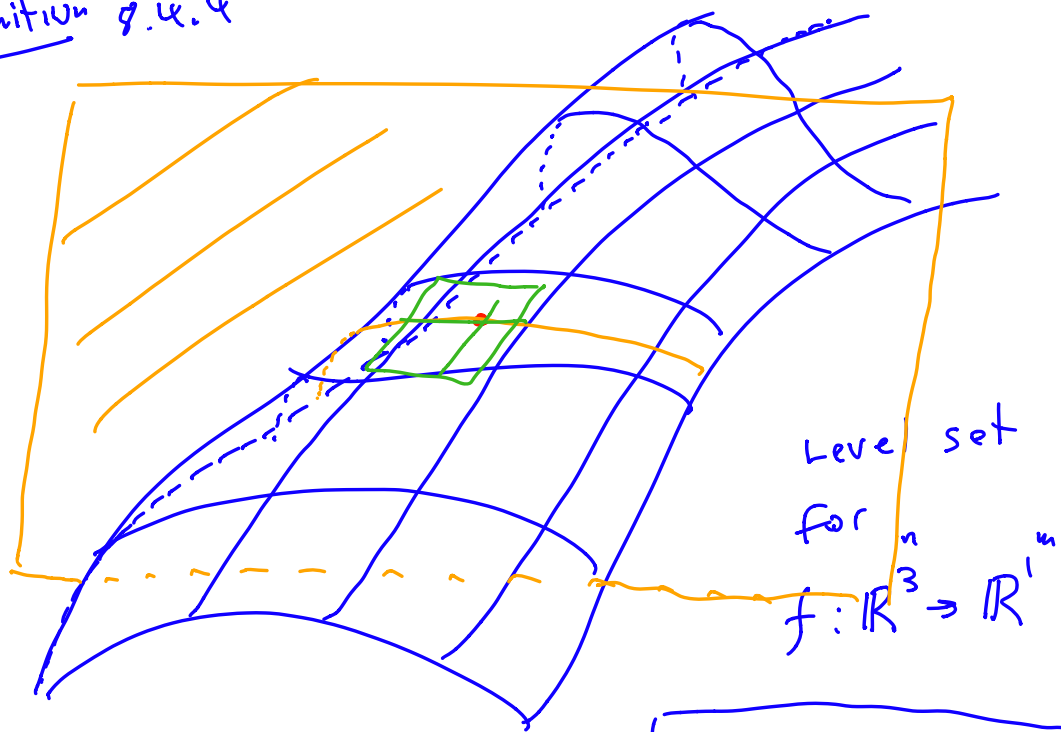
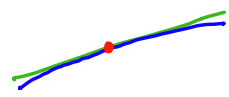
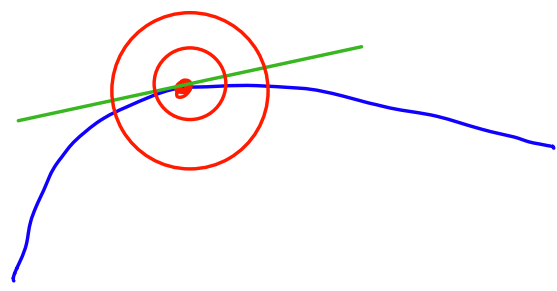


Definition 9.4.4



Level sets are
 $n-m$ dimensional



$$\textcircled{1} \quad \begin{array}{c} \text{vector in } \mathbb{R}^n \\ \swarrow \\ A x = b \leftarrow \text{in } \mathbb{R}^m \leftarrow \\ \uparrow \\ m \times n \text{ matrix} \end{array}$$

$$\textcircled{2} \quad F(x+h) - F(x) = A_x h + o(h)$$

$$\textcircled{3} \quad \int_E g(x) JF(x) d\mathcal{L}_x^n =$$

$$\int_{F(E)} \left(\int_{F^{-1}(y) \cap E} g(x) d\mathcal{H}_x^k \right) d\mathcal{H}_y^m$$

where

$$k = \max(n-m, 0)$$

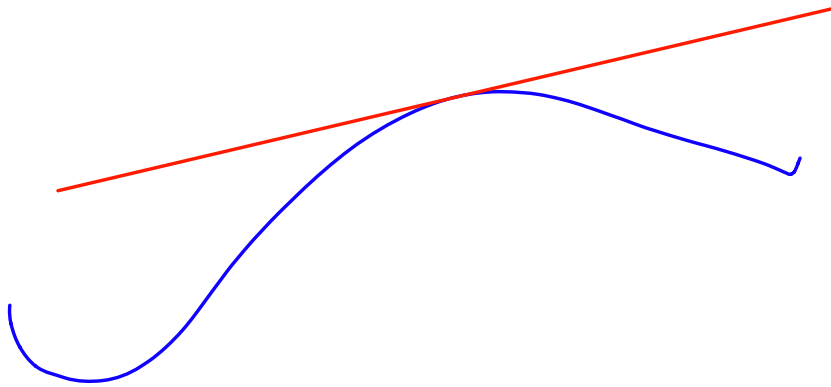
$$F: \mathbb{R}^n \rightarrow \mathbb{R}^m$$

F is smooth

$$JF \equiv \sqrt{\det(DF \circ DF^T)} \\ \text{or} \\ \sqrt{\det(DF^T \circ DF)}$$

$$[DF^T][DF] \leftarrow n \leq m$$

$$[DF][DF]^T \leftarrow n \geq m$$

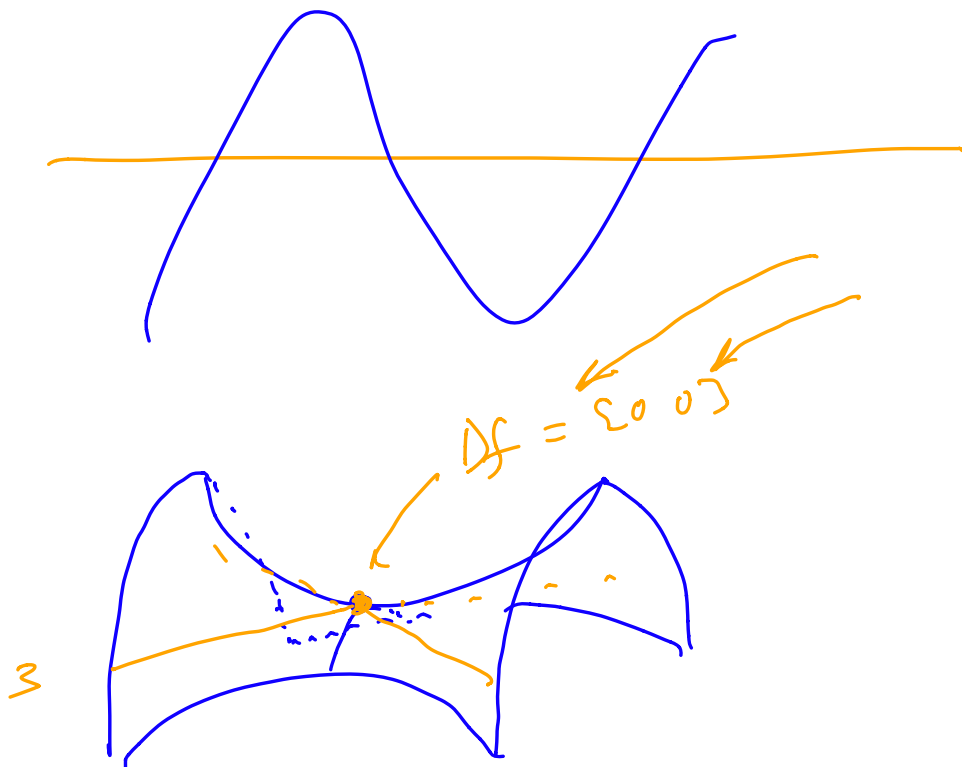


$$\begin{array}{c} \downarrow \downarrow \downarrow \downarrow \downarrow \\ \rightarrow \begin{bmatrix} x & x & x & x & x & x \\ x & x & x & x & x & x \\ x & x & x & x & x & x \end{bmatrix} : \mathbb{R}^6 \rightarrow \mathbb{R}^3 \end{array}$$

→ rank = # of ind columns (or rows)

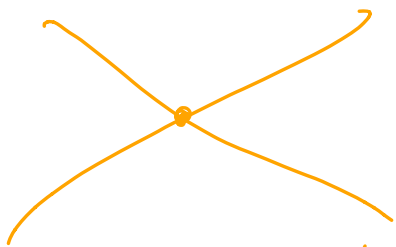
$$\rightarrow \text{rank} = \dim(\text{span}(\text{columns}))$$

$$\rightarrow \text{rank} = \dim(\text{domain}) - \dim(\text{kernel})$$



$$f: \mathbb{R}^2 \rightarrow \mathbb{R}$$

$$[x \ x]$$

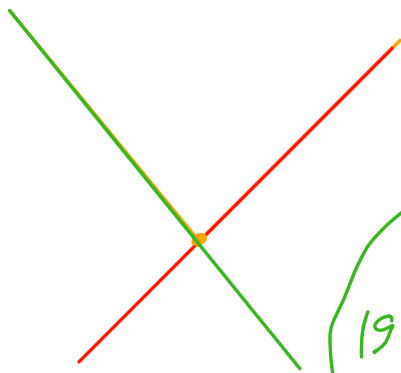


$$X_3 \equiv \{ x \mid f(x) = 3 \}$$

$$F(x+h) - F(x) = A_x h + o(h)$$

$$\underline{F(x+h) - F(x) - A_x h = \underline{g(h)}}$$

$$\frac{|g(h)|}{|h|} \xrightarrow{|h| \rightarrow 0} 0$$



$$|g(h)| \leq C|h|$$

 $O(h)$

$$\frac{|g(h)|}{|h|} \leq C < \infty$$

for $h \in B(0, \varepsilon)$
 $\varepsilon > 0$