$\Delta u = \hat{\nabla} F$.

$$\frac{dF(u)}{du}(h) = -2\int_\Omega \Delta u \cdot h \, du$$

F : functions $\to$ Reals

$$F(u+h) = F(u) + Ah + \underbrace{g(h)}_{o(h)}$$

$F : \mathbb{R} \to \mathbb{R}$

$$\Updownarrow$$

$$F(u+h) - F(u) - Ah = \underbrace{g(h)}_{\text{residual}}$$

$L(\alpha v + \beta w)$

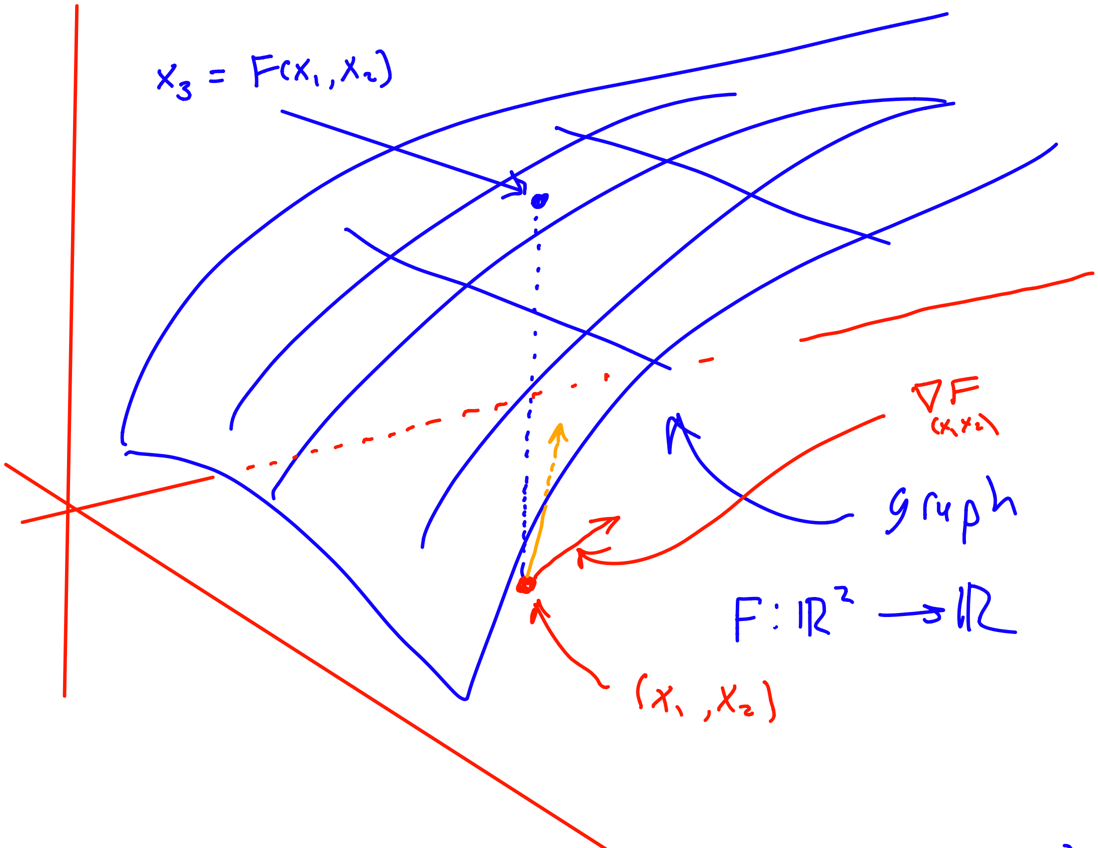
$= \alpha L(v) + \beta L(w)$

$$\frac{|F(u+h) - F(u) - Ah|}{|h|} \to 0$$

$\underline{|h| \to 0}$

$$\frac{|h|^2}{|h|} = |h|$$

$\underline{\mathbb{R}^3 \to \mathbb{R}}$ $\leftarrow$



column vector

$$F(x+h) \approx F(x) + \underbrace{\nabla F \cdot h}$$

row vector

$$\underbrace{\begin{bmatrix} | \\ v \\ | \end{bmatrix}}_{\in \mathbb{R}^n} \longrightarrow \mathbb{R}$$

$$\begin{array}{cc} & n \\ 1 & \begin{bmatrix} \end{bmatrix} \end{array}$$

$$F(x+h) \approx F(x) + \langle \hat{\nabla} F, h \rangle$$

vector that makes this work

inner product

$$\nabla F = \langle \hat{\nabla} F, \cdot \rangle$$

side note

$$V \quad , \quad V^*$$

vector space

space of continuous linear maps from V to $\mathbb{R}$

$|x| = \sqrt{\langle x, x \rangle}$

(Hilbert space)

if $\omega \in V^*$ $\exists$ u_ω $\ni$

$$\omega(v) = \langle u_\omega, v \rangle \quad \forall \; v \in V$$

$u : R \to \mathbb{R}$

$\to [a,b] \times [c,d]$

$\to [0,1] \times [0,1]$

measurement $\to d = u + \eta$

$$F(u) = \int_R |\nabla u \cdot \nabla u| \, d\mu + \lambda \int_R |u - d|^2 \, d\mu$$

$$\nabla F(h) = -2 \int \Delta u \cdot h \, d\mu + \lambda \int 2(u-d) \, h \, d\mu$$

$$\nabla F(h) = \int \left[-2 \Delta u + 2 \lambda (u-d) \right] h \, d\mu$$

$$\Downarrow$$

$$\hat{\nabla} F = -2 \Delta u + 2 \lambda (u-d)$$

$$\boxed{\begin{aligned} u_{i+1} &= u_i + \alpha \left(- \hat{\nabla} F \right) \\ &= u_i + \alpha \left(2 \Delta u_i - 2 \lambda (u_i - d) \right) \end{aligned}}$$

old $\longrightarrow \displaystyle\int |\nabla u|^2 \, d\mu + \lambda \int |u-d|^2 \, d\mu$

ROF $\longrightarrow \displaystyle\int |\nabla u| \, d\mu + \lambda \int |u-d|^2 \, d\mu$

L^1TV $\longrightarrow \displaystyle\int |\nabla u| \, d\mu + \lambda \int |u-d| \, d\mu$